PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Dynamical Models of the Middle Atmosphere for Tracer Studies

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Dynamical models of the middle atmosphere for tracer studies

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The distribution of ozone and other tracers depends on chemical factors and on the atmospheric circulations, which themselves depend, through heat sources and sinks, on the ozone amount. Examples of the sensitivity of ozone to variations in the dynamics are given from a two dimensional model. The total ozone depends greatly on the heating rate in the equatorial lower stratosphere; changes of tens of milli atmosphere-centimetres† arise from alteration of only a few tens of kelvins per day in heating. The effects of variations in momentum flux and mixing-surface slope are also discussed.

The current ability to model dynamics for tracer studies is reviewed. Only three dimensional models provide a consistent formulation; they simulate many observed atmospheric phenomena broadly but not in detail. However, the amount of computing needed precludes all but the simplest treatment of chemistry. Two dimensional models offer a valuable compromise. Although these are rather empirical and not adequately based on sound physical principles, good agreement can be found in practice between observed and modelled ozone, winds and temperatures.

The calculation of model-dependent meridional circulations in two dimensional models is shown to be important, for which a parametrization of the momentum flux is an essential prerequisite. Some observations are given that suggest that a parametrization based on potential vorticity fluxes may be possible. This would be consistent with the usual formulation of tracer transport.

1. Introduction

The spatial abundance of ozone has been known since the very earliest measurements to depend not only on photochemistry, but also on redistribution by the winds (Dobson *et al.* 1928). This became apparent as the latitudinal distribution of total ozone was determined (figure 1), because, contrary to expectations based on photochemistry alone, the minimum column densities are in low latitudes while the maxima occur in high latitudes.

The need for quantiative models of these motion systems has assumed some urgency if potential threats to the ozone layer from manmade pollution are to be properly evaluated. Calculations of the photochemical equilibrium state give equatorial column densities slightly over twice those observed, from which it can be loosely inferred that changes of, say, 10% in the velocities can produce changes in the ozone column densities of between 5 and 10%. This is comparable to the predicted decreases in ozone due to continued release of chlorofluoromethanes at the 1973 rate calculated in simple models (N.A.S. 1976). Accordingly, it is important to understand the extent to which the motion systems themselves depend on the ozone amount (see, for example, Pyle 1978). We are faced with a complicated feedback loop, for the motions are driven to a large extent by the stratospheric heat sources and sinks, the source (the absorption of solar radiation at the stratopause) depending strongly on the ozone amount. The sink (the emission of long wave radiation) is also weakly ozone-dependent. Moreover, stratospheric motions are partially forced by planetary waves propagating from the troposphere and

† 1 milli atmosphere-centimetre (hereafter abbreviated 1m atm-cm) = 10^5 m at s.t.p.

the details of this motion, too, depend on the radiative damping which is intricately connected with the ozone chemistry (Lindzen & Goody 1965; Blake & Lindzen 1973).

This paper reviews the progress that has been made in modelling the dynamical side of these processes, dealing exclusively with models which have been developed primarily for studies of ozone and other tracers. It makes no attempt to cover the extensive literature on analytic (linearized) studies of planetary propagation (see, for example, Holton 1975) and wave–mean flow interaction (see, for example, McIntyre & Andrews 1976).

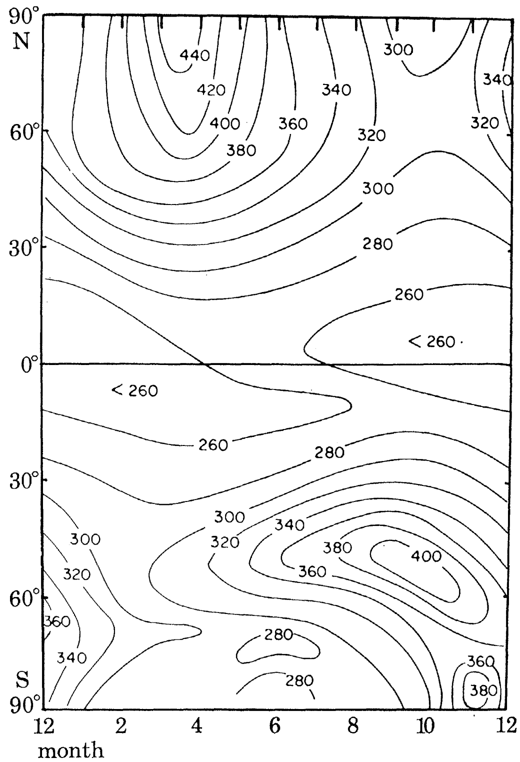

FIGURE 1. The observed zonally averaged distribution of total ozone (m atm-cm) (Dütsch 1971). The low latitude minima and the hemispheric differences immediately reveal the importance of the dynamics of transport processes.

2. THREE DIMENSIONAL MODELS

(a) *Highly truncated spectral models*

The atmosphere is a three dimensional system so it is natural to assume that anything less than a three dimensional model is unlikely to be satisfactory. In practice, however, finite resources mean that some feedback mechanisms need to be curtailed and the price of a three dimensional treatment will be a gross simplification of the physical processes at work. It may not be possible to achieve a detailed treatment of the chemistry or the run may not extend long enough for a quasi-steady state to be reached.

Because the stratospheric circulation is dominated by the largest scale waves, spectral methods with severely truncated spectra appear to offer an economical representation. Instead of being represented at each horizontal level by values at regularly spaced intervals, as in the more conventional grid point methods, the fields are represented by the leading coefficients in their

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spherical harmonic decomposition. A very large fraction of the variance of stratospheric motion is accounted for by only a few coefficients, so that large savings in computation appear possible. The situation is not straightforward, however, because the waves are forced in the troposphere. The tropospheric forcing mechanism is in part an interaction between the zonal mean wind and the relevant Fourier component of the orography and heating pattern and in part is a non-linear interaction between higher wavenumbers. Both mechanisms involve the higher wave-numbers; the latter obviously, and the former because the energy of the zonal mean wind is maintained by transfer from the baroclinic eddies, which are predominantly of wavenumbers around 6. Consequently, the severe truncation may not be possible for the tropospheric forcing.

A further saving in computer time can be made by using the quasi-geostrophic equations rather than the primitive equations. The main predictive equation here is the vorticity equation in which the vorticities and winds are obtained from the height fields by using what is essentially a geostrophic approximation. This prevents gravity waves from developing and permits considerably longer time-steps than can be taken in primitive equation models. Lorenz (1960) has shown how to develop energetically consistent versions of the quasi-geostrophic equations and his formulation provides the starting point of most spectral models.

One of the first attempts at a spectral model using spherical harmonics was that of Clark (1970), whose main aim was to investigate the ozone budget and energetics of the wintertime circulation (see also Peng 1965). To minimize computing time he employed a rather severe truncation, retaining only the zonal mean field and zonal wavenumbers 1, 2, 3 and 6. Moreover he retained only those harmonics of wind, temperature and ozone which are symmetric across the equator, and with only the lowest order modes in N–S direction. This gave about 20 degrees of freedom for each variable in the horizontal and he employed a six-layer model in the vertical with winds held at 750, 350, 150, 62.5, 15 and 2.5 mbar.† His ozone photochemistry was a linearized form of the Chapman scheme with rate coefficients somewhat arbitrarily adjusted to give moderate agreement with the observed distributions. The model was run for a total of 250 days with a constant (winter) solar declination starting from radiative-photochemical equilibrium.

Notwithstanding its obvious limitations, the overall behaviour of this model was sufficiently encouraging for it to be extended by Cunnold *et al.* (1975) to produce probably the most comprehensive three dimensional joint dynamical, chemical model of the stratosphere to date, their aim being the study of the possible effects on the ozone layer of supersonic airliners.

They removed the requirement of symmetry across the equator and also included *all* zonal wavenumbers less than or equal to 6. Hence the treatment of the troposphere is much improved, although the representation is still very crude compared with that of a typical tropospheric general circulation model. The global mean temperature is specified in the model and only departures from it are calculated. This greatly reduces the computational effort required to deal with convective overturning, latent heat release and surface energy balance in the troposphere. Symmetry between hemispheres was assumed for the distribution of land form and sea-surface, so that the model is unable to reproduce the observed interhemispheric differences in ozone distribution.

Their treatment of ozone photochemistry is also a great improvement upon that of Clark, mainly by not linearizing the photochemistry and by the inclusion of the catalytic destruction of

† 1 mbar = 10^2 Pa.

ozone by the oxides of nitrogen. However, ozone is the only species calculated explicitly in the model, the distribution of NO_x being specified as tables to the model, at first independently of latitude (Cunnold *et al.* 1975), then later (Alyea *et al.* 1975) with the use of distributions obtained from the steady-state two dimensional model of Hesstvedt (1974*a*) (see § 3 (*a*) below).

Two dimensional distributions of NO_x appropriate to both the natural and polluted stratosphere were employed so as to assess the likely effects of supersonic passenger aircraft on the ozone layer. The ozone behaviour in the 'natural' run showed excellent realism though the maximum values were about 10% higher than those in figure 1, while the minimum values were too low by about 5%.

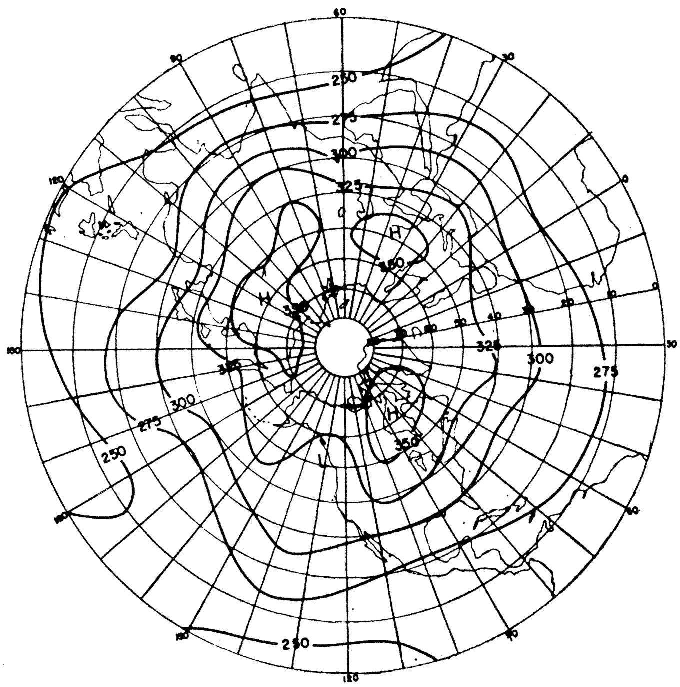

FIGURE 2. Horizontal distribution of total ozone (m atm-cm) for winter calculated in a quasi-geostrophic spectral model (Cunnold *et al.* 1975).

Figure 2 shows the distribution of total ozone for winter, taken from Cunnold *et al.* (1975). Although the polar values in this particular run are considerably lower than observed, owing largely to the latitudinally constant NO_x profile, the displacement of the largest values towards Siberia is in broad agreement with the observations compiled by Wu (1973). The major waves thus appear to be well represented although some difficulties with excessive wavenumber 6 amplitudes in the stratosphere are reported.

The wind and temperature field predicted by the model have a fair measure of agreement with observations. In particular, the strength of the jets in the upper stratosphere are quite similar to the observations of Murgatroyd (1969) and Newell *et al.* (1969). However, the modelled temperature gradient does not reverse above the stratosphere so the wind maxima are not attained within the modelled region. This is probably because vertical velocities are inhibited by the upper boundary conditions so that the adiabatic cooling and warming above the summer and winter polar stratopauses respectively are correspondingly underestimated.

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The 'polluted' stratosphere case corresponds to an injection from aircraft of 1.8×10^6 metric tons of NO_x per year. On a global average the ozone reduction amounted to about 12 %, but the high latitude changes were nearer 20 % with corresponding smaller low latitude changes.

It is ironical that under the pressure of the C.I.A.P. schedule coupled with the need to economize on computer time, a full three dimensional distribution of NO_x could not be calculated, because the principal supposed advantage of three dimensional models for pollution studies is thereby lost, namely that the transports of all constituents, natural or pollution, are calculated in a consistent and realistic way. Some indications of the potential importance of this are given by comparing the latudinal distribution of total ozone in the run, where NO_x is independent of latitude, to that given by the two dimensional model. The pole–equator gradient in spring is increased by 60 Dobson units by this modification. While the effects on the longitudinal structure cannot be expected to be so great, they are probably appreciable. Nonetheless this is probably the most dynamically complete model with realistic nitrogen oxide catalytic destruction of ozone yet to have been produced.

Studies of the dynamics of two quasi-geostrophic spectral models with high truncation have emphasized the importance of the feedback loops outlined in § 1 above. Trenberth (1973) identified the cause of minor warmings in his model with horizontal transports of heat by the transient eddies, the mechanism being in broad accord with the theory put forward by Matsuno (1971). The transience was related to transience in the vertical energy flux (correlations between pressure and vertical velocity) which, in turn, Trenberth attributed to the tropospheric index cycle associated with 'beating' between the stationary and travelling disturbances. A number of interesting relations between the stratospheric warmings and tropospheric events, such as blocking and index cycles, are thus highlighted. On the other hand, the strength of both the zonal mean circulation and the eddies exhibit considerable sensitivity to the approximation adopted for the long wave cooling (Ramanthan & Grose 1978; Chen & Ramanthan 1978).

All the models mentioned above show departures from the details of the observed atmosphere, but many features are successfully reproduced. They will continue to provide powerful tools as advances in computer power allow the restrictions, such as the level of truncation or the number of species carried explicitly, to be relaxed.

(*b*) *Primitive equation models*

All the models mentioned so far have been based on the quasi-geostrophic equations because of the reduction in computation afforded by the longer time steps permitted by this approximation. As the approximation requires small Rossby numbers, the flow in the equatorial region may be misrepresented. In particular, the Kelvin and mixed gravity–Rossby waves cannot be treated properly. This in turn means, on current thinking, that the quasi-biennial oscillation and the semi-annual wave at 50 km will not be reproduced (Holton 1975; Hirota 1978). The interhemispheric exchange of stratospheric material may be poorly estimated in consequence. Moreover, it is possible that even the large scale extratropical disturbances may be misrepresented by the quasi-geostrophic equation (Dickinson 1968; Holton 1976), including the tropopause-folding process that is responsible for cross-tropopause transports. Accordingly, great interest attaches to comparing models with and without the quasi-geotrophic simplification. Such a direct comparison is not yet possible as the models differ in many more respects than simply in which dynamical approximations were adopted.

A major series of studies of the stratosphere has been undertaken by Manabe, Hunt, Mahlman and others using primitive equation models based on the G.F.D.L. general circulation model. The earlier studies (e.g. Hunt & Manabe 1968; Hunt 1969) concerned hemispheric versions of the model, often with 'perpetual winter' conditions, but these restrictions have been removed more recently (Manabe & Mahlman 1976; Mahlman & Moxim 1978).

Manabe & Mahlman (1976) give the result of a 3.5 year run of the model with seasonal variation of solar radiation, (prescribed) sea surface temperatures, and cloud amounts. The model has 11 vertical levels, the top level being at 10 mbar; the vertical velocity (strictly speaking, ω in σ-coordinates) is set to zero at $p = 0$; the horizontal resolution is approximately 265 km; the tropospheric hydrological cycle is computed in detail and a very full radiative computation is performed. This uses the modelled water vapour amounts but all other radiatively active constituents, including ozone, are prescribed as tables of latitude, height and date.

The overall behaviour of the model is summarized in table 1, taken from Mahlman & Moxim (1978). This shows a high level of achievement but it also indicates that, even with the very complete treatment of the physical and dynamical processes attained in this model, it is possible to show considerable departure from the observed atmospheric state. In particular, the polar night jet in the model and the associated equator–pole temperature difference are two or three times stronger in the model than in reality. This may be an effect of the lid-like boundary condition or of the limited vertical resolution at the upper level. Moreover, the polar night jet is not sufficiently separated from the subtropical jet in the Northern Hemisphere winter. This is linked to the strength of the mid-latitude descending motion.

Table 1. Summary of the behaviour of the model of Manabe & Mahlman (1976)

category	comments
zonal mean temperature	Very good below 38 mbar. Too cold in polar night middle stratosphere by 20 K. Northern Hemisphere. (N.H.) midwinter stratospheric warming not simulated
tropopause	Agrees well with observed poleward-downward slopes and appears at the proper altitudes.
zonal mean wind	N.H. subtropical jet slightly weaker than observed. Polar night stratospheric jet too strong by a factor of 2. Tropical stratospheric easterlies too strong by 15 m s^{-1}; quasi-biennial oscillation not simulated. Seasonal variation of zonal wind qualitatively satisfactory.
meridional circulation	Qualitatively correct: 3-cell troposphere, 2-cell N.H. stratosphere, 3-cell S.H. stratosphere. Midstratosphere flow from summer to winter hemisphere.
eddy kinetic energy	Somewhat too small in midlatitude troposphere. Good in N.H. stratosphere. Very large seasonal and interhemispheric variations in stratosphere.
stationary disturbances	Structure very good, amplitude somewhat large, particularly in N.H. middle stratosphere winter.
transient disturbances	Cyclonic disturbances somewhat weak in midlatitude troposphere. Transient long waves somewhat weak in midlatitude stratosphere. Tropical transient waves qualitatively correct but vortex disturbances too large in scale.
precipitation	General qualitative agreement with observation. Average continental precipitation somewhat too high. Too little precipitation in southern United States and southwestern China.

One of the major aims of this simulation is to investigate interhemispheric differences. Accordingly, the topography and sea surface temperatures specified in the Southern Hemisphere are appropriate to that hemisphere. Thus the level of planetary wave activity and the consequent forcing of the mean circulation are different in each hemisphere. This is clearly brought out in figure 3. The modelled eddy kinetic energy at 10 mbar in both the standing and the transient eddies is larger in the northern winter than in the southern; the interhemispheric difference in the standing wave energy being a factor of over ten while the difference in

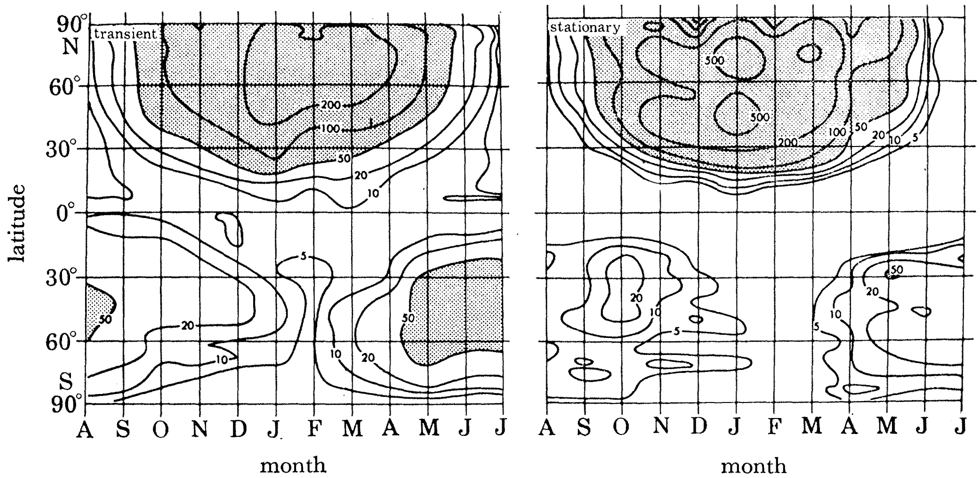

Figure 3. Latitude–time distribution of two components of eddy kinetic energy (J kg^{-1}) at 10 mbar in a three dimensional primitive equation model (Manabe & Mahlman 1976). The dominance of standing eddies in the Northern Hemisphere and transient in the Southern Hemisphere is an observed feature.

transient wave energy is nearer to four. In the Northern Hemisphere, standing wave energy is more than two times larger than transient, whereas in the Southern Hemisphere the transient wave energy is larger than the standing. The difference in eddy activity between the hemispheres is greater than is shown by the observations, but the relative importance of the transient disturbances in the Southern Hemisphere has been remarked upon by many authors (e.g. Harwood 1975; Leovy & Webster 1976; Hartmann 1976).

Mahlman & Moxim (1978) have used the winds from this model to simulate the behaviour of radioactive fallout. The initial injection was centred on 65 mbar at 36° N, 180° E and the subsequent spread of tracer was followed in detail over the next four simulated years (the winds from 1 year of the dynamical simulation being repeated four times). The results show that certain basic features such as the interhemispheric exchange are handled well, so that during the first year cross-equatorial transports reduce the ratio of material in the Northern Hemisphere to that in the Southern to about four, further reducing it to about two over the next 3 years, in general accord with the behaviour of $^{14}CO_2$ from 1963 to 1966 as found by Telegadas (1971). Stratospheric residence times of relevance to one-dimensional transport models based on the modelled distribution increase with the height of the centre of mass of tracer by 0.35 year per kilometre, being about 1.6 years at 20 km.

By October of the first year, a large cancellation is established between the zonal mean rates of change of tracer mixing ratio produced by eddies and by mean motions (see figure 4) as is found by most models. This cancellation is bound to arise in steady conditions for any property whose Lagrangian time derivative is near to zero (Andrews & McIntyre 1978). Under these circumstances, averages along the deformed streamlines of the Lagrangian motion show zero rates of change. Transience disturbs the average position of the streamlines in the meridional plane so that the cancellation in the zonal means is no longer exact. Thus the cancellation may be somewhat exaggerated in this particular model since it does not produce significant sudden warmings.

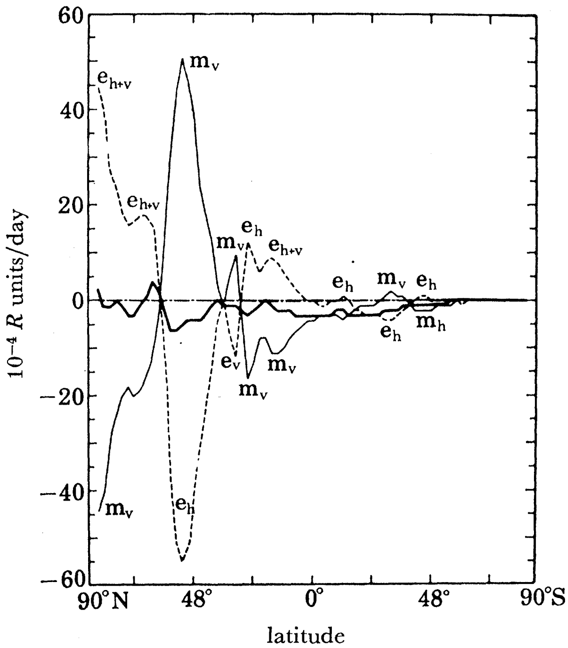

FIGURE 4. Mechanisms leading to the change at 65 mbar during October of the zonal mean mixing ratio of a tracer, R, in the same model as in figure 3 (10^{-4} R units/day) (Mahlman & Moxim 1978). Note the large cancellation between the mean and eddy contributions. The labels on the extrema denote mean (m) and eddy (e) contributions, with the subscript showing if the horizontal (h) or vertical (v) transport dominates. ——, Net tendency; ----, eddies; ——, meridional circulation; -·-·-·, 'filling' + 'diffusion'.

When averaged over the globe, transports across the model tropopause are approximately an order of magnitude more by the eddy transport than by the zonal mean motion. However, at a given latitude the two effects are of similar magnitude. This arises because the eddy fluxes are usually downward whereas the mean flux depends on the local sense of the zonal mean vertical velocity. The calculated downward eddy flux through the tropopause is strongest near 50–60° N in March–May, predominantly associated with developing extratropical cyclones, while the strongest downward zonal mean fluxes occur near 35–45° N during December–February.

A group working in the U.K. Meteorological Office have also made a substantial study by using a primitive equation stratospheric model (Newson 1974; C.O.M.E.S.A. 1975).

The model has 13 σ-levels and was derived, mainly by increasing the vertical resolution, from that of Corby *et al.* (1972). Eight levels are normally in the stratosphere, the highest being near 44 km; the horizontal resolution is close to 300 km; tropospheric physical processes are treated in considerable detail; a smoothed version of the Earth's topography is assumed; and sea surface temperatures vary as seasonal climatological values, but land temperatures adjust according to radiative and convective heat fluxes.

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This work is of particular interest since realistic sudden warmings occur in the winter stratosphere. Figure 5 shows the changes in zonal mean wind between day 70 and day 90 of an earlier run of this model (Newson 1974) in which only one hemisphere was treated and 'perpetual January' conditions were adopted. On day 70, the stratospheric polar night jet is too strong by a factor of perhaps two, and the distinction between the polar night jet and the subtropical jet is insufficiently marked. This appears not to be a result of maintaining the January conditions for too long, as the seasonally varying model behaves similarly. After day 78 a dramatic change occurred; by day 90 the temperature at the pole had risen by 60 K at 40 km and the zonal mean wind reversed (figure 5*b*). The polar night vortex began to re-form on about day 100, but even by day 130 it was not as intense as it had been before day 78.

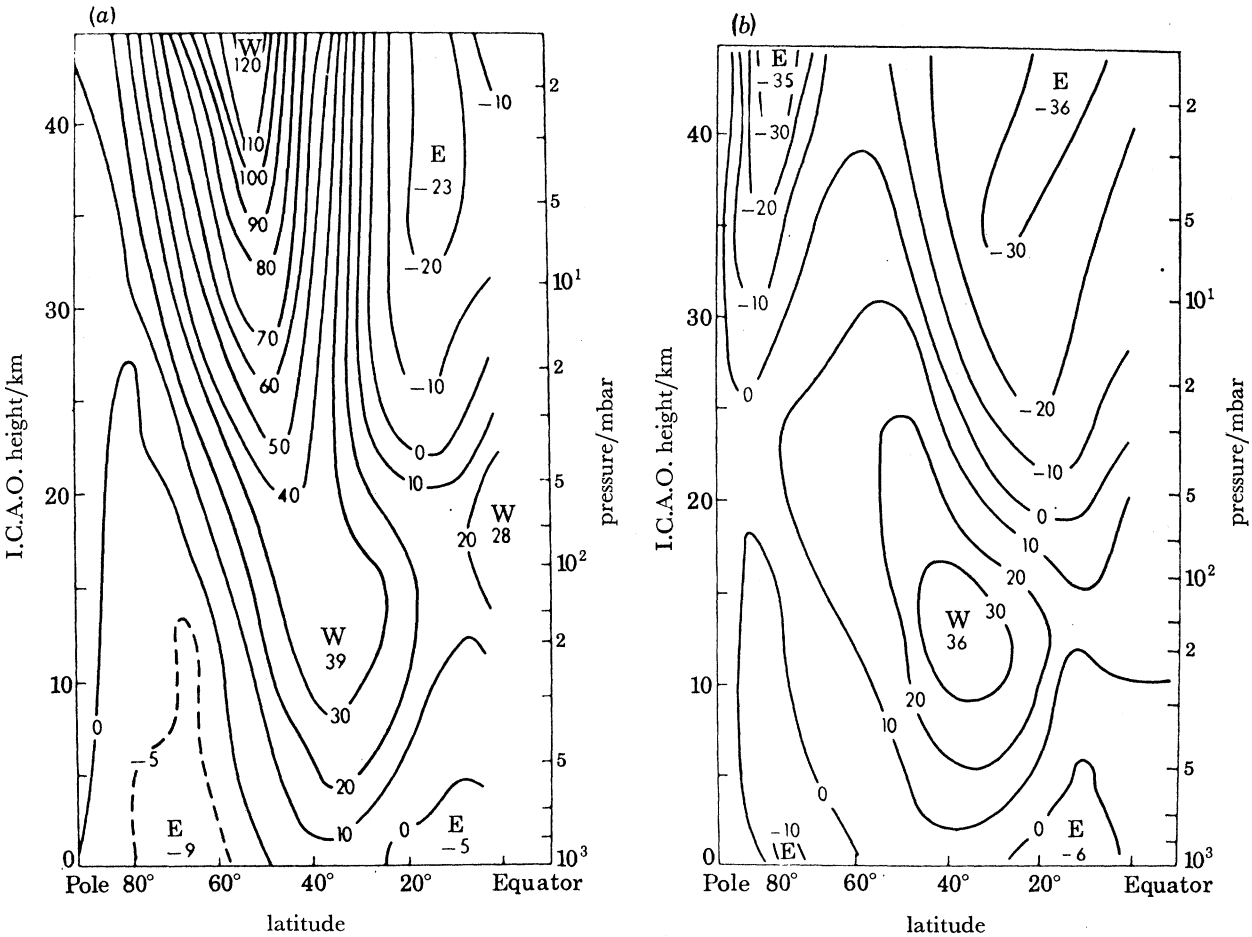


FIGURE 5. A sudden warming simulated in a three dimensional primitive equation model. Zonal mean winds ($m\ s^{-1}$) on (*a*) day 70, before the onset, (*b*) day 90, 12 days after the onset of warming (Newson 1974).

It is probable that the major factor in the success of this model in producing a major warming compared with the G.F.D.L. model is the superior resolution of the upper stratosphere. In the G.F.D.L. model, vertical velocities in the upper level must be distorted by the presence of the 'lid' boundary conditions. Furthermore, radiative damping of the planetary waves and critical level interactions will be better handled by the C.O.M.E.S.A. model.

So far the model has been used to study the spread of tracers that were inert or had very simple ozone-like sources and sinks. Runs with a more realistic treatment of ozone are awaited with great interest.

3. Two dimensional models

(a) *Averaging the photochemical and continuity equations*

The large amounts of computing time required to perform the dynamical calculations in three dimensional models has usually meant that only a very simplified treatment can be made of the chemical processes. For instance, Cunnold *et al.* (1975) state that their model requires 40 s central processor time to simulate 1 model day on an I.B.M. 360/90, while Newson's hemispheric calculation required 9 min per day on an IBM 360/195. Both were thereby forced to adopt very simple photochemical schemes in which only one tracer was explicitly calculated. To improve the modelling of the chemistry, therefore, many workers have sought to reduce the amount of computation by constructing two dimensional models, calculating only the zonal mean of the relevant variables as functions of latitude and height.

Two difficulties arise immediately, namely how to calculate the relevant chemical production rates and how to calculate the meridional transports. Consider first the chemical processes. Let s be the local value of some variable and let $\bar{s}$ denote the zonal mean of s. Let s' be the departure of s from the zonal mean, $s' = s - \bar{s}$. If we now consider two reacting substances having mixing ratios a and b, then the zonally averaged rate of the reaction is proportional to $\overline{(ab)}$.

This may be split up thus, $$\overline{ab} = \bar{a}\bar{b} + \overline{a'b'},$$

into a contribution by the mean state $\bar{a}\bar{b}$, and a contribution by the 'anomalies' or 'eddies' $\overline{a'b'}$.

The eddy contribution involves quantities that are not explicit model variables and is usually simply ignored. It is not difficult, however, to conceive of circumstances where this might matter. For instance, if at a certain height and latitude all the 'a' substance is found in air parcels that are rising and all the 'b' substance is in sinking air, then the two substances never mix and the true reaction rate $\overline{(ab)}$ is zero. However, the usual formulation gives instead a finite reaction rate $\bar{a}\bar{b}$ (Tuck 1979).

Photochemical processes likewise present difficulties because the transmission of radiation through the atmosphere is not a linear function of absorber amount. Again it is usually assumed that the zonally averaged rate of photodissociation is that which would obtain for the zonal mean absorber amounts.

The difficulties with transport are analogous. The zonally averaged continuity equations for the explicitly modelled gases require knowledge of meridional transports. If the gas S has mixing ratio s, then the northward transport of S involves expressions of the form $\overline{vs}$ where v is the northward velocity. This may be split up into two terms involving transport by the zonal mean winds ($\bar{v}\bar{s}$) and the eddies $(\overline{v's'})$

$$\overline{vs} = \bar{v}\bar{s} + \overline{v's'}.$$

The greater problem is posed by the eddy transport term $\overline{v's'}$ since s' and v' are not model variables.

The technique almost universally adopted is a parametrization in terms of the zonal mean fields, proposed by Reed & German (1965). They introduced eddy diffusion coefficients K_{yy}, K_{yz} such that

$$\overline{v's'} = -K_{yy}\,\partial\bar{s}/\partial y - K_{yz}\,\partial\bar{s}/\partial z,$$

y being distance northwards and z being height.

A mixing-length hypothesis provides some justification for this formulation and further suggests that all substances having similar mixing processes will have the same diffusion coefficients. In principle, this means that measurements of the fluxes and distributions of two suitable quantities would determine the value of K_{yy} and K_{yz}. However, for the tracers that are most easily measured such as ozone and potential temperature the problem is ill-conditioned because of the high correlation between them (Mahlman & Maxim (1978) gives an example of the build-up of such correlations). Accordingly, less direct methods must be used. Once more the favoured method was proposed by Reed & German, again based on the mixing-length idea, though more tentatively: K_{yy} is assumed to be proportional to $\overline{(v')^2}$, which is readily observed. Adoption of a suitable constant of proportionality completely determines K_{yy} and the measurement of transports and distribution of only one tracer now suffices to find K_{yz}.

Eddy transport in the vertical cannot be neglected and here two new coefficients are introduced by

$$\overline{w's'} = -K_{zy}\,\partial\bar{s}/\partial y - K_{zz}\,\partial\bar{s}/\partial z,$$

where w is the vertical velocity.

Mixing-length theory gives $K_{zy} = K_{yz}$, so one measurement of a vertical tracer flux distribution in principle determines K_{zz}. However, vertical fluxes are in general very poorly known and such direct estimates are not available. Again, mixing-length theory provides an estimate, this time involving the means and variances of the slope of the mixing motions. Even here the relevant quantities are not available and most estimates are little more than intelligent guesses of about the correct order of magnitude.

In spite of all the uncertainties involved – whether mixing-length ideas can adequately represent the processes; whether the same *K*s are suitable for all substances; what the values of K really should be – it can be said in favour of this method that it works surprisingly well, as we shall see below, and as yet no alternative method is available. Fabian & Libby (1974) tested several combinations of *K*s and mean meridional circulations by computing the spread of a radioactive tracer in a two dimensional model and comparing the results against observations. They obtained the most consistent results by combining a mean meridional circulation obtained from Gudiksen *et al.* (1968) with a set of *K*s deduced by Luther (1974), who followed closely Reed & German's method based on heat flux data.

It will be noticed that the K approach is most suited to treatment of the natural stratosphere. The addition of pollution can be expected to induce changes in the eddies either directly by altering the radiative damping or indirectly by altering the zonal mean state through which they must propagate. In terms of mixing-length theory this implies changes in $\overline{v'^2}$ and hence K_{yy}, and in the mean slope of the mixing paths (which turns out to be K_{yz}/K_{yy}). Likewise, the inter-annual variation in atmospheric state should imply the use of different *K*s for each year but the sophistication involved in altering the *K*s from year to year probably exceeds the physical basis on which they are founded.

It should be noted in passing that all the problems associated with averaging, which have been mentioned with two dimensional models, occur even more chronically in one dimensional models. The usefulness of all these models lies not in their ability to predict, say, the effects of introducing pollution, but rather in their ability to reveal some of the possible major mechanisms and feedbacks at work.

Transport by the mean motion $\bar{v}\bar{s}$ is relatively straightforward, but even here care is needed not to attempt modelling experiments beyond the scope of the model, for while it is reasonable to adopt observed values of mean circulation for studies of the unpolluted stratosphere, the same is not true if any pollution affects the radiatively active constituents (such as ozone). Besides the direct effect through alteration of the zonal mean radiative heating, an indirect effect can be anticipated through modification of the eddy heat and momentum fluxes consequent upon alteration of the planetary waves through changes to their radiative damping. As yet we have little knowledge of the likely importance of such effects.

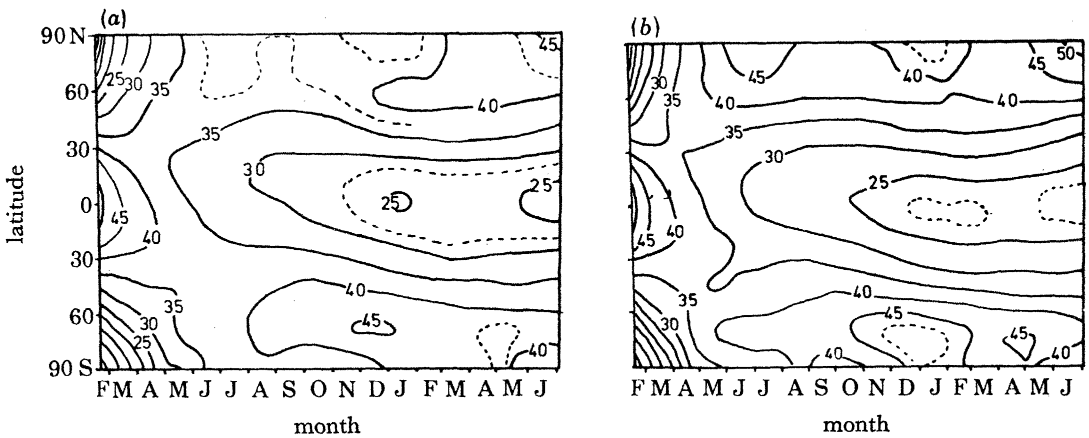


FIGURE 6. The sensitivity of ozone in a two dimensional model to the diffusion coefficients. Total ozone (10^{-2} atm-cm) calculated (*a*) by using Luther's (1974) *K*-values, (*b*) with K_{yz} doubled in mid- and high latitudes (equivalent to doubling the slope of the mixing surfaces) (Harwood 1977).

Most models take the values of the transport parameters ($\bar{v}$, $\bar{w}$ and the eddy coefficients) as external variables supplied to the program as suitable tables from estimates based on observations or three dimensional models. However, recognizing that these transport parameters are but poorly known, several investigators adjust them to produce a good agreement between modelled and observed ozone amounts. Selecting two examples from many, the model of Brasseur & Bertin (C.O.V.O.S. 1976) utilized rather crude spatial distributions of the *K*s with magnitudes adjusted to give realistic O_3 distributions, while the U.K. Meteorological Office's 2-D Comesa model had an adjustment made to the mean meridional circulation above 25 km after two simulated years so as to improve the total ozone distribution (C.O.M.E.S.A. 1975).

Some indication of the magnitude of the response to changes in the diffusion coefficients is given by figure 6 from Harwood (1977). The calculations were performed in a two dimensional general circulation model with a linear approximation to the photochemical source and sink of ozone. The only difference between the two runs lies in the values of K_{yz} adopted in mid and high latitudes in the mid-stratosphere. Doubling this coefficient increased the high latitude ozone amounts by about 10%, associated with a lowering of the height of maximum ozone concentration at the poles. The equatorial values decreased by about a similar fraction, even though K_{yz} was unchanged there.

While it may be that adjustments to obtain a better agreement with observations are necessary because of inadequate knowledge of the transport parameters, there are obvious dangers in the practice. In particular, one may be attempting to use the transports to correct for what is really a deficiency in the chemical scheme. It is salutary in this respect to consider the growth during the last two decades in the number of reactions thought necessary to explain the ozone cycle.

A considerable saving in computation can often be made for some purposes by seeking steady-state solutions with solar declination held constant. It is perhaps surprising that this leads to reasonable distributions for ozone, since the stratospheric residence times of many months appear too long to allow an approach to steadiness within a season. Nonetheless, Rao & Christie (1973) and Hesstvedt (1974*a*) found moderately good distributions of ozone in steady-state models. The reason for their success probably lies in the large cancellatoins between mean and eddy transports already referred to (§ 2.2 above and figure 4): only a small change in the ozone distribution is needed to the steady-state distribution to produce the observed secular rates of change. In this context it is noteworthy that Hesstvedt (1974*b*) reports that a time dependent version of his model gave markedly better results than the steady-state version. Comparison is hindered, however, by the use of different *Ks* in the two versions. Although it is often claimed (see, for example, Widhopf & Taylor 1975) that adoption of mean velocities and eddy coefficients deduced from different data sets will lead to a gross disturbance of the near cancellation between mean and eddy flux convergence, in practice the near cancellation is found. Harwood & Pyle (1977) contend that this is to be expected, because in *K*-theory the eddies are a diffusive system which, since K_{yy} is typically $10^6\ \mathrm{m^2\,s^{-1}}$, has a diffusive timescale of only 10 days for distances of 1000 km. Consequently, transient effects are very rapidly damped out, leaving a quasi-stationary state in which the near cancellation is found. The resulting tracer distribution may not be very realistic, of course, but the cancellation is unavoidable.

(*b*) *Model dependent mean circulations*

The importance of the mean circulation to the ozone amounts has led to several models in which it is calculated in terms of the model state. To exemplify some of the methods, we will outline the technique of Harwood & Pyle (1975).

The relevant dynamical equations are:

(1) the zonal mean momentum equation

$$\frac{\partial(\bar{\tau}F)}{\partial t}+\frac{\partial(\overline{v\tau}F)}{\partial y}+\frac{\partial(\overline{w\tau}F)}{\partial \eta} = H, \tag{1}$$

(2) the zonal mean thermodynamic equation

$$\frac{\partial(\bar{\theta}F)}{\partial t}+\frac{\partial(\overline{v\theta}F)}{\partial y}+\frac{\partial(\overline{w\theta}F)}{\partial \eta} = Q+\bar{q}F, \tag{2}$$

and

(3) the continuity equation

$$\frac{\partial(F\bar{v})}{\partial y}+\frac{\partial(F\bar{w})}{\partial \eta} = 0, \tag{3}$$

where $F = (p/p_0 \cos\theta)$, p is pressure, $p_0 = 1000$ mbar, y = distance north, $\eta = \ln(p_0/p)$, $w = \mathrm{D}\eta/\mathrm{D}t$, θ = potential temperature, τ = angular momentum = $(u+a\Omega\cos\lambda)\ a\cos\lambda$,

8-2

with u = westerly wind speed, v = northward wind speed, a = radius of the Earth, $\Omega = 2\pi$ radians/sidereal day, λ = latitude. q is the rate of change of potential temperature due to diabatic processes.

$$H = -\frac{\partial(\overline{v'\tau'F})}{\partial y} - \frac{\partial(\overline{w'\tau'F})}{\partial \eta}; \quad Q = -\frac{\partial(\overline{v'\theta'F})}{\partial y} - \frac{\partial(\overline{w'\theta'F})}{\partial \eta}.$$

As with the tracer equations, the major difficulty is presented by the need to calculate the eddy terms H and Q in terms of the zonal mean state. Q is amenable to K-theory treatment; indeed, as we have seen, the Ks are usually calculated on this basis from observed potential temperature fluxes. H is less tractable and will de discussed later (§ 3 (b)).

The equations so far do not constitute a complete set, the y-momentum equation having been so far omitted. The use of the complete y-equation leads to a kind of primitive equation model which contains gravity waves and thus requires short time steps to satisfy the C.F.D.L. criterion. Moreover, untrapped gravity waves are expected to have velocity amplitude proportional to $\rho^{-\frac{1}{2}}$, which increases by two orders of magnitude between the ground and mesopause, so that small gravity wave disturbances excited in the lower layers may dominate the upper layers. Primitive equation methods are not ruled out, however, and Holton (1976) has used a slightly approximate form of equation (1)–(3) with the complete y-momentum equation to investigate the sudden warming phenomena.

The gravity waves may be filtered out by employing the thermal wind equation

$$\frac{\partial \bar{\tau}}{\partial \eta} = -\frac{R}{2\Omega \tan \lambda}\left(\frac{p}{p_0}\right)^K \frac{\partial \bar{\theta}}{\partial y}. \tag{4}$$

White (1979) has shown that there is a slight energetic inconsistency introduced by this form of equation (4) which can be corrected by including a nonlinear term in (4) or making suitable approximations in equation (1). However, the spurious energy source is trivial compared with natural energy exchanges in the model.

Introducing a stream function ψ such that equation (3) becomes

$$v = -\frac{1}{F}\frac{\partial \psi}{\partial \eta}, \quad w = +\frac{1}{F}\frac{\partial \psi}{\partial y},$$

and eliminating the time derivatives from (1) and (2) by using (4), gives

$$\pounds(\bar{\tau}, \bar{\theta}; \psi) = R_1(H) + R_2(Q) + R_2(qF), \tag{5}$$

where $\pounds$ is a linear second order partial differential operator for ψ which is elliptic if the zonal mean state is inertially stable. The coefficients are functions of the mean state $\bar{\tau}$, $\bar{\theta}$. R_1 and R_2 are differential operators, involving principally horizontal gradients of heating effects, R_2, and vertical gradients of eddy momentum flux convergence, R_1. The right-hand sides thus arise from processes which tend to disturb thermal wind balance. Equation (5) determines that circulation which is necessary to maintain the balance, and is solved for ψ at each time step and the value substituted in (1) and (2), which are then used to step forward in time.

Rao's (1973) method has many similarities with the scheme just described, but also several major differences: he holds the solar declination constant and finds the steady solution for $\bar{\tau}$, $\bar{\theta}$ and $\overline{[O_3]}$; he uses a constant value of the coriolis parameter between latitudes 0 and 25° in each Hemisphere in his version of equation (4) so as to ensure that the inertial stability criterion is satisfied. Consequently, it is probable that the cross-equatorial winds and interhemispheric exchange of material is incorrectly estimated. A further difference lies in the parametrization of momentum fluxes discussed in § 4.

The success of schemes outlined above can be judged from figures 7–9 taken from Harwood & Pyle (1977) and Pyle (1976). In the case portrayed, the diabatic processes q were calculated explicitly in terms of model temperatures and absorber amount using a cooling-to-space approximation for long wave radiation around 50 km. Tropospheric heat sources due to radiation and latent heat release were specified from climatology. Q, the eddy heat flux convergence, was calculated by K-theory. The vertical momentum fluxes were calculated from $\overline{w'\tau'} = -k_{\eta\eta}(\partial\bar{\tau}/\partial\eta)$, but the horizontal fluxes were specified from observations. The ozone

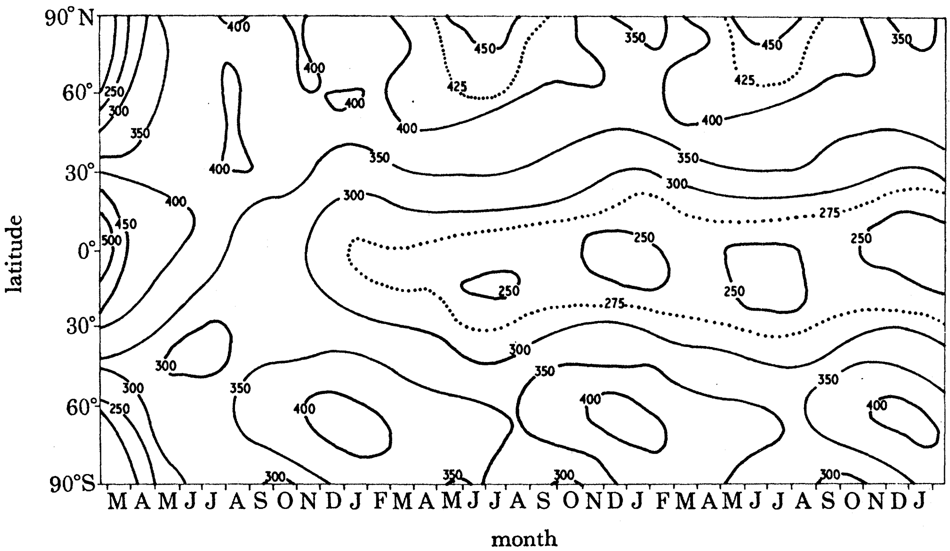


FIGURE 7. The distribution of total ozone (m atm-cm) calculated in a two dimensional time dependent model (Harwood & Pyle 1977). The starting values correspond to photochemical equilibrium. The interhemispheric variations (cf. figure 1) arise from asymmetries in the adopted momentum fluxes.

chemistry was linearized so that the creation or destruction rate is simply proportional to the departure from the equilibrium value. The constants of proportionality (reciprocals of the relaxation times) and photochemical equilibrium concentrations were specified to the program in the form of tables as functions of latitude, pressure and season, having been calculated offline from a reaction scheme that includes the catalytic cycles of NO_x and HO_x. There was a sink of ozone in the model troposphere: the ozone concentration at the bottom level is maintained at zero, thus resembling rapid destruction at the ground. Each model day required 2.5 s of computing time on a CDC 7600.

The latitude–time section of total ozone for the run is given in figure 7. On the starting day (19 February) the ozone takes its spring photochemical steady-state concentration and it is nearly a year before a periodically reproducible distribution is established. At the end of the run there are small year-to-year differences.

Once the model has settled down, many of the observed features of the total ozone distribution are reproduced. The magnitude of the total ozone amount is quite good. The equatorial minimum is modelled well with closed minima of less than 250 Dobson units. The latitudinal variation is about 2–3 months out of phase with the observed behaviour with a minimum in the model in the Northern Hemisphere in December, and an observed minimum at the beginning of October. This is perhaps not unreasonable since the momentum fluxes and K-coefficients are obtained from 1 and 5 years' data respectively and should not necessarily reproduce an ozone distribution averaged over a much longer period of time (since fluctuations from year to year are not insignificant (Dütsch 1971)). Experience with a later version of the model in which the NO_x was calculated as a model variable produced the high latitude ozone maximum in spring, implying that the main cause of the late maxima and minima is the linearization of the photochemical scheme. In particular, the destruction rate is known to be underestimated by this approximation.

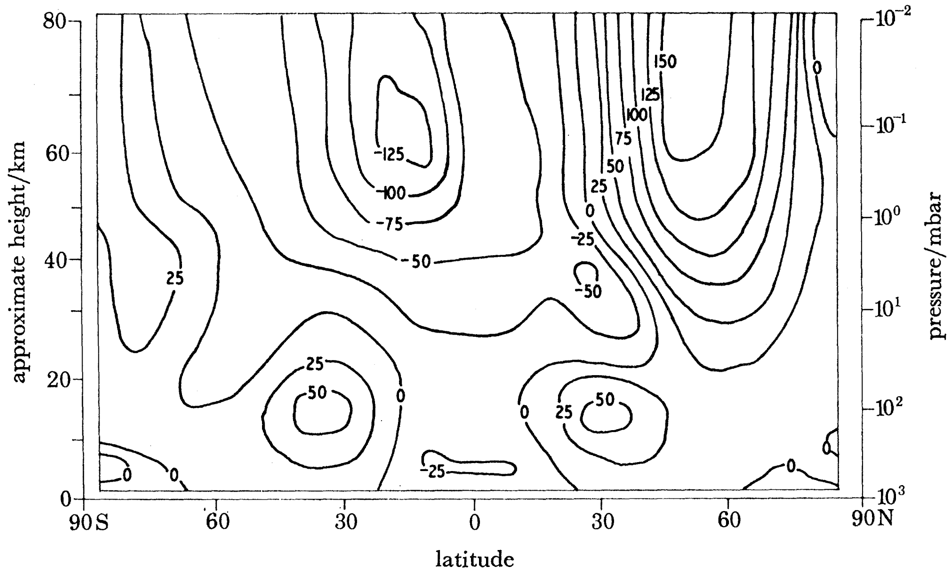


FIGURE 8. Zonal mean winds for 22 December of the second year of the simulation shown in figure 7 (Pyle 1976).

Figure 7 points at once to the importance of the dynamical feedback since the principal asymmetry between the Hemispheres comes from the horizontal eddy momentum fluxes specified in the stratosphere: the diffusion coefficients and the parameters from which the radiation is calculated are seasonally symmetric between the Hemispheres. The amount of ozone in mid and high latitudes of the Northern Hemisphere is 5–12 % higher than that in the Southern Hemisphere. It follows that any prediction of pollution-induced changes of this order of magnitude from a model without dynamics cannot be considered reliable unless it can also be shown that the adjustment of the eddy momentum flux to the new final state is considerably smaller than the currently observed difference between the Hemispheres.

Figure 8 shows the mean zonal wind in the model on day 671, which corresponds to 22nd December. The west–east wind component reproduces the gross features of the observed winds in both the troposphere and stratosphere, but with important differences of detail. In the troposphere the subtropical jets are separated by a region of tropical easterlies, and there are regions of surface easterlies at the poles. The latitudes of the polar night jet in the stratosphere and the easterly jet are in good agreement with the observations. The extension of the stratospheric easterly régime into the winter hemisphere is also a feature of observations. The easterly jet at 30° N and 40 km is not shown on most cross sections. However, Kats (1968) has reported similar features, though of much lesser intensity. The principal jets in figure 8 are larger than the observed circulation by a factor of between 1.5 and 2. It follows that the horizontal temperature gradients are also exaggerated in the model, although the equatorial temperatures are in good accord with observations (Pyle 1976). The excess temperatures at the summer stratopause are probably due to an overestimate of the ozone and hence absorption of solar radiation in that region, but the excessively cold winter pole is not understood.

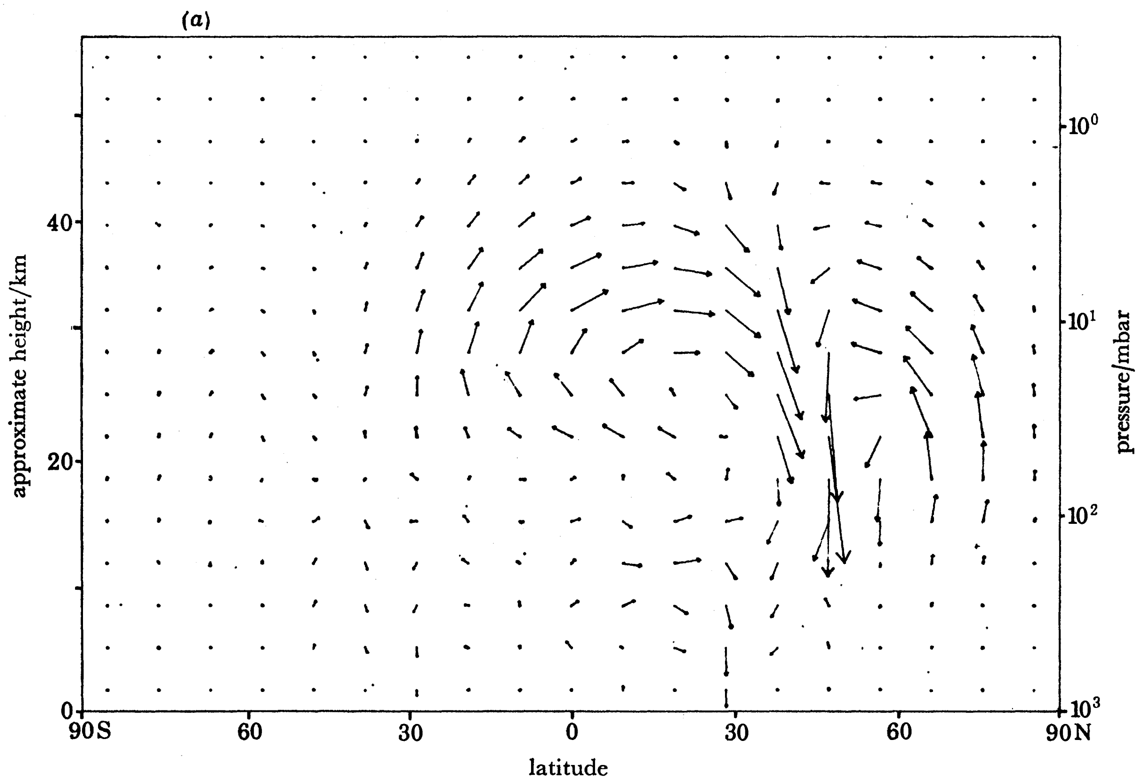
The mean meridional circulation of the model and the modelled meridional distribution of ozone have been shown (Pyle 1976) to agree sufficiently well with the values of Vincent (1968) and Dütsch (1971), respectively, for confidence to be placed in the values of modelled ozone transport shown in figure 9, partitioned into mean and eddy fluxes. It is of importance to note that the main *vertical* fluxes are carried by the mean motion and not by the eddies: the processes usually represented by a vertical 'eddy' diffusion in one dimensional global models are what two dimensional modellers regard as the mean motion with planetary dimensions, if the fluxes are averaged horizontally over a hemisphere.

This is emphasized in figure 10 from Harwood (1977), which shows the vertical flux of ozone, integrated horizontally over the whole of the Northern Hemisphere. The eddy flux contribution is tiny. Note that the hemispheric average mid-stratospheric flux is downward in the winter but upward in the summer. It follows that the hemispherically averaged ozone flux is sometimes up- and sometimes down-gradient. Similar behaviour can be expected for other constituents also. Indeed, it is easy to see that, given the right latitudinal distribution of material, the hemispheric-average flux at certain levels could always be directed against the hemispheric-average gradient. This further underlines the need for caution in evaluating the results of one dimensional models.

Figure 9 demonstrates that the amount of ozone in high latitudes is greatly dependent on the strength of the sinking motions in mid-latitudes. Again, the wisdom of calculating model-dependent mean motions in pollution studies is indicated.

A striking example of the sensitivity of the ozone amounts to dynamical influences is provided by the response to a small change in the radiation field of the lower stratosphere. The 'control run' is that described above, which produced the ozone distribution already seen in figure 7. In this model the lower stratosphere was forced to be in radiative equilibrium by setting the radiative heating and cooling to zero between about 12 and 24 km at all latitudes. Figure 11 shows the result of changing this assumption. The principal change made from the 'control' run was in the region 12–24 km where the radiative equilibrium condition was replaced by heating rates from computations by Rodgers (1967). This introduces heating rates of a few tenths of a degree per day in low latitudes and cooling of comparable magnitude in high latitudes in winter. (There was also a small change to the bottom boundary condition which has been shown to be insignificant.) The influence of this change in radiation on the mean circulation was marked, there being a far stronger ascent in the stratospheric Hadley cell. This produced great changes in the ozone amount as may be seen from figure 11.

The implication for our present purpose is that any alteration to the radiation budget of the lower stratosphere by only a few tenths of a degree Kelvin per day could have quite large effects on the ozone distribution. This sensitivity to the lower stratospheric heating rates may present difficulties for studies of pollution-induced perturbations, as the total heating is a comparatively small residual of two large numbers. One of these depends to a large extent on the amount of ozone above. It is quite likely, therefore, that the feedback loop, ozone $\rightarrow$ heating $\rightarrow$ mean motions $\rightarrow$ ozone, must be taken into account for reliable computation of pollution effects.



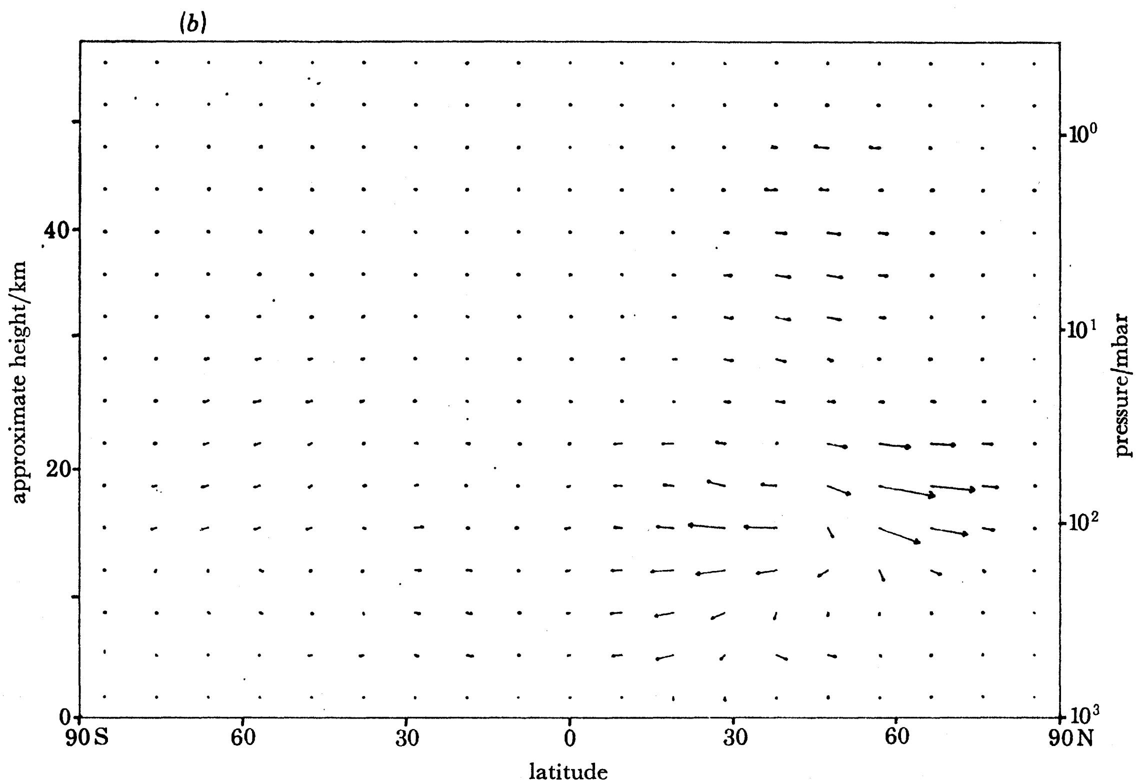


FIGURE 9 *a* and *b*. For description see opposite.

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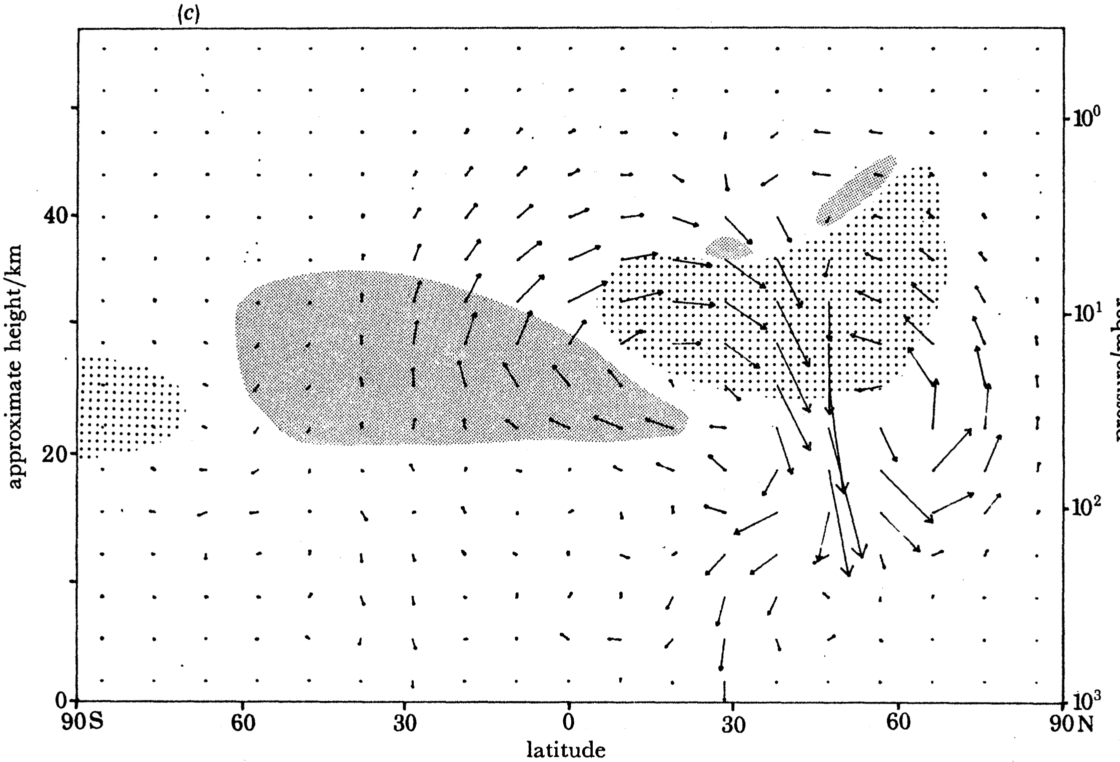

FIGURE 9. Meridional flux of ozone for the same case as figure 8 (*a*) due to mean motions, (*b*) due to eddies, (*c*) total. Fine shading denotes a photochemical source greater than 3×10^9 cm^{-3} day^{-1}. Coarse shading denotes a photochemical sink greater than 3×10^9 cm^{-3} day^{-1} (Harwood & Pyle 1977).

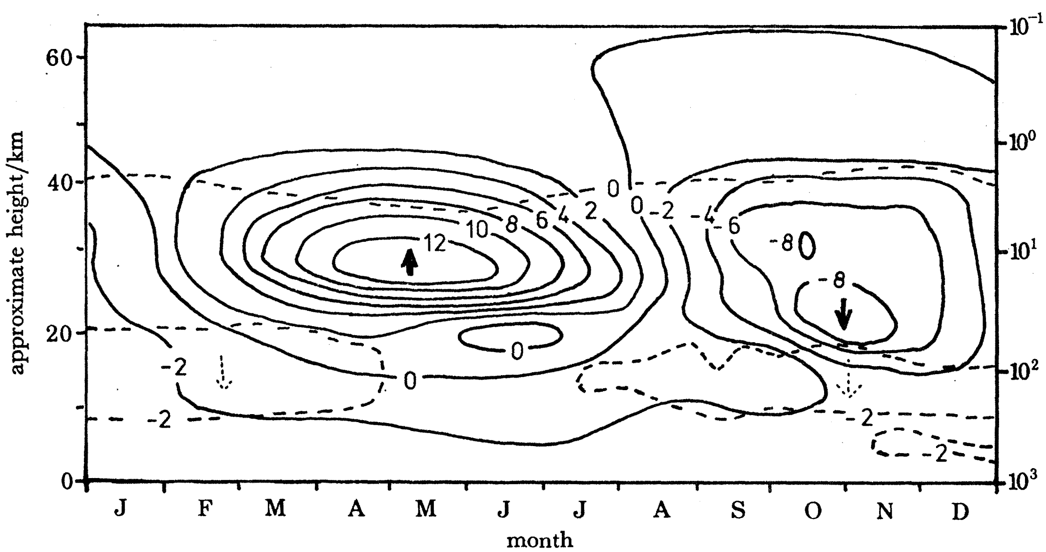

FIGURE 10. Northern Hemisphere average vertical ozone flux (10^{10} mol cm^{-2} s^{-1}) in a model similar to that in figure 7. ——, Mean motions; – – –, eddy motions.

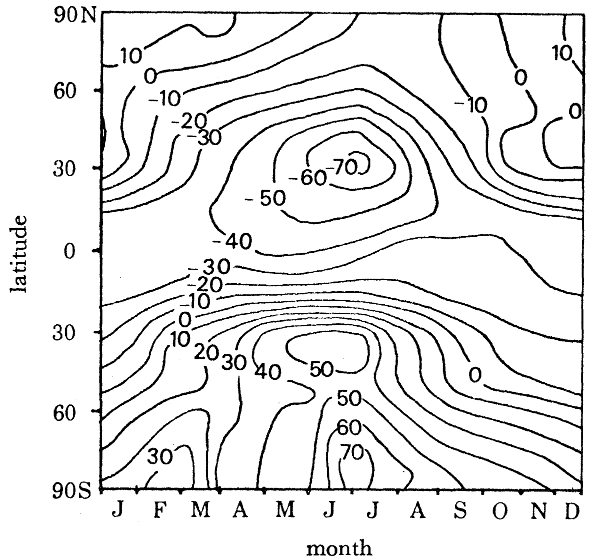

FIGURE 11. Change in total ozone (m atm-cm) caused by changing the lower stratospheric heating rate.

4. Modelling the momentum flux

The previous considerations have pointed to the importance of calculating the mean circulation explicitly. To do this it is necessary to determine the eddy momentum flux. Figure 12 provides an example of the dependence of total ozone on the momentum flux. The two curves show the latitudinal distributions of total ozone in two runs of the model described above, which differed only in the value of the eddy momentum fluxes prescribed in the upper stratosphere. These values have been compiled by Dr A. Crane of the Clarendon Laboratory, Oxford, based on temperature observations from the selective chopper radiometer on the Nimbus 5 satellite. The mid-latitude ozone amounts change by about 5 %. The true sensitivity may be underestimated by this experiment as the momentum fluxes in the lower stratosphere were unaltered between the two runs.

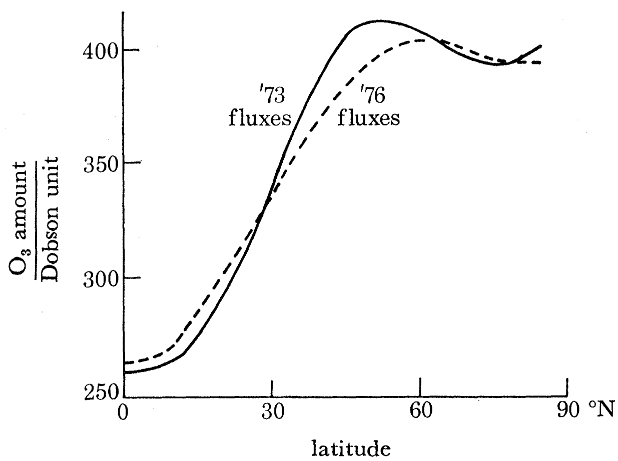

FIGURE 12. Sensitivity of the ozone amount to the momentum fluxes. Latitudinal distribution of total ozone (m atm-cm) on 1 April for two runs differing only in the year of the observed momentum fluxes used in finding the zonal mean circulations.

As yet, no satisfactory method of calculating the momentum fluxes from the model state has been proved in a model, although some attempts have been made. Rao (1973) uses a parametrization of the momentum flux proposed by Williams & Davies (1965) in which the fluxes are proportional to the pole–equator temperature gradient. This parametrization was proposed for tropospheric modelling on the assumption that the biggest contribution to the fluxes is made by the baroclinic eddies. However, even in the troposphere the relevant factors are considerably more complicated than is here implied (Green 1970; White 1977) while the present theories of planetary wave propagation to the stratosphere (Holton 1975) give no encouragement to the belief that the Williams & Davies formulation is realistic above the tropopause.

Another method which has been proposed for the tropospheric eddy momentum fluxes is that of Saltzman & Vernekar (1968). Assuming that the trough ridge systems are produced baroclinically but then tilted in the latitude–longitude direction by barotropic processes, they obtain a parametrization of the flux convergence dependent on the tilting time. While the method appeared to give encouraging results in the troposphere it is unlikely that it provides a suitable scheme for stratospheric eddies.

The success of the theories of planetary-wave propagation (Charney & Drazin 1961; Matsuno 1970; Simmons 1974) offer the possibility that an acceptable configuration of standing planetary eddies could be found every few time steps from which the momentum (also heat fluxes) could be calculated, or even the time evolution of the eddies calculated. In essence this method is used by Matsuno (1971) and Holton (1976) in their studies of the sudden warming phenomenon. Unfortunately, great simplifications were necessary, mainly the linearization of the wave equations. Moreover, the method involves inversion of an elliptic operator, which is computationally expensive, so that a severely truncated spectral model may prove cheaper. Schoeberl & Strobel (1978) have attempted to construct a steady state quasi-geostrophic model based on this method but were forced to abandon it because of numerical instabilities that developed when they used realistic amplitudes for the forcing of the wavenumber 1 wave. Instead, they made use of a Rayleigh friction in which the eddy flux convergence is made proportional to the zonal mean wind. This is simply a mathematical device necessary to prevent the occurrence of strong wind speeds in the absence of a proper eddy formulation; it has no physical justification whatsoever in the stratosphere.

A desirable property of any parametrization of the momentum fluxes is that it should be consistent with the other formulations. Green (1970) has outlined a procedure, which can be adapted to stratospheric use, that has this property. At the heart of the method is a formula relating the eddy momentum flux to the fluxes of potential temperature and potential vorticity. The main adaption needed for the stratosphere is to use an approximate form of the full Ertel potential vorticity rather than the quasi-geostrophic form used by Green. Since potential temperature and potential vorticity are both quasi-conservative properties, at least in the lower stratosphere, it should be possible to treat them by K-theory. Indeed, we have already seen that quite acceptable values can be obtained for heat transport. In conclusion we show some evidence that K-theory is capable of modelling the potential vorticity flux.

Figure 13 shows the observed distribution and flux of potential vorticity at 30 mbar for November 1976–April 1977. In mid-latitudes the flux is directed towards the pole, which is predominantly up the gradient on the constant pressure surface. In low latitudes the flux is towards the equator. Similar results have been obtained for this height for December 1970–January 1971 and for October 1975–January 1976. Hartmann (1977) found rather similar

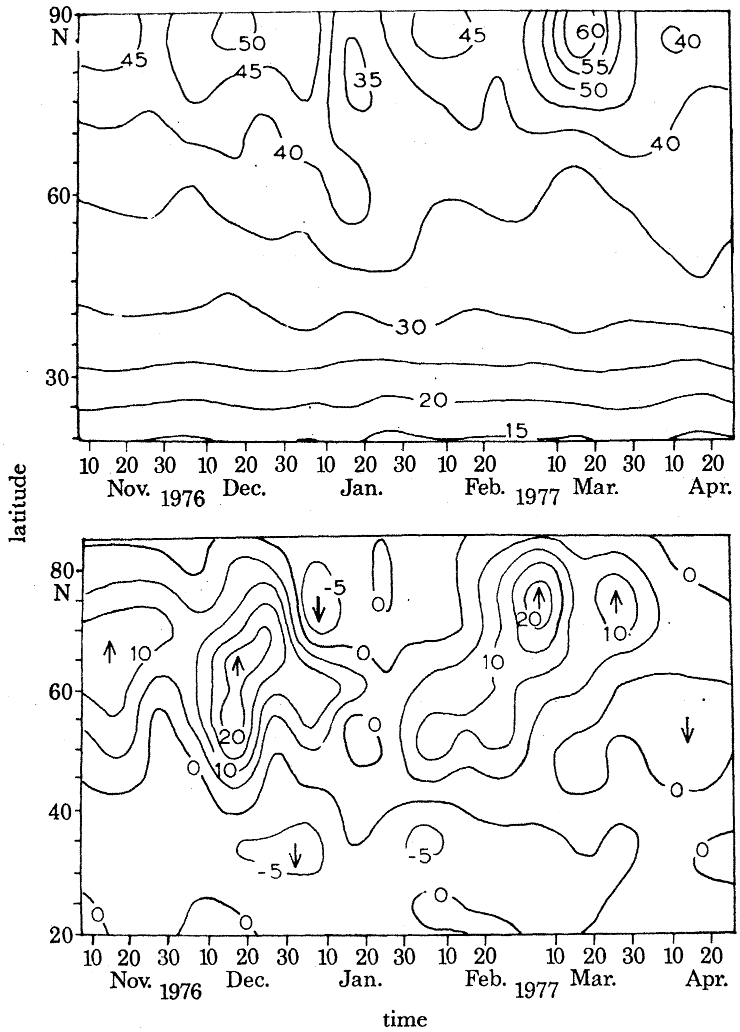

FIGURE 13. Observed time latitude section at 30 mbar of (*a*) potential vorticity, Q, (*b*) northward eddy flux of potential vorticity, $\overline{v'Q'}\cos\phi$, where ϕ = latitude. Note the counter-gradient flux at high latitudes.

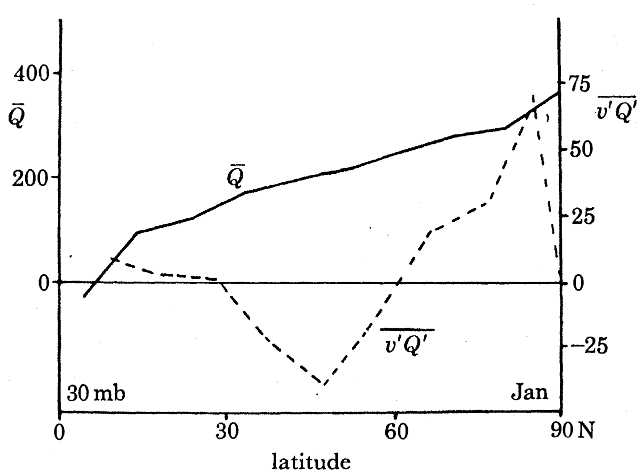

FIGURE 14. Distribution of potential vorticity $\bar{Q}$ and the northward eddy flux $\overline{v'Q'}$, both non-dimensionalized calculated for the C.I.R.A. 65 January atmosphere using the diffusion coefficients of Luther (1974).

transports in the Southern Hemisphere. Note that the 'up-gradient' transport of high latitudes must arise because the mixing paths (assuming the concept is applicable) slope more steeply than the surfaces of constant potential vorticity which slope poleward and downward.

The final figure (14) shows the potential vorticity flux deduced from a January distribution in a standard atmosphere (C.I.R.A. 1965) using the K values of Luther. The qualitative structure is about right but the magnitude of the mid-latitude poleward flux is too small. This suggests that the implied slope of the mixing paths is too shallow in this region. It is significant that Fabian & Libby (1974) found that the main discrepancy between observed and modelled tracer spread using these diffusion coefficients also pointed to an underestimation of this slope.

In view of the large arbitrariness in the usual method of finding diffusion coefficients, it seems probable that a set of coefficients could be found which would give realistic fluxes of both heat and potential vorticity thus allowing the momentum flux and hence the mean circulation to be calculated from the model state. Although such a model would be susceptible to the same criticism as all K-theories, it would have a powerful consistency between the assumptions adopted about the chemical transports and the dynamics.

5. Concluding remarks

The interactions between tracer gases, radiation and dynamics in the stratosphere are clearly too complex for a full and consistent treatment to be achieved with the computing power available in the foreseeable future. Transport processes depend on three dimensional systems with lifetimes of days. As yet, no sound physical basis exists for parametrizing the transports, so the only way to guarantee consistency is to perform a calculation capable of resolving this time and space scale. However, the stratospheric residence times are such that, even if other chemical factors are held constant, a simulation needs to extend for 2 or 3 years for a quasi-steady state to be reached. Such a calculation is at the limit of present computing power. However, the complete treatment of the chemistry requires many species and reactions to be included; it requires runs extending for decades because of species with reaction times of years, and simultaneously requires very short time steps to be taken to accommodate quick reactions or rapid adaptations to sunrise and sunset.

Understanding will grow only by considering a spectrum of models each of which tackles only part of the problem, from sophisticated three dimensional dynamical models with very crude chemistry on the one hand to multi-species chemical models with very crude dynamics on the other. Two dimensional models have an important role to play, lying in the middle of this spectrum. A pressing need here is to 'close' the calculation of the meridional circulation by parametrizing the momentum flux. The remarks in the previous section give some hope that this is possible. Another pressing need in two dimensional models is for a more consistent diffusion theory. Here we may anticipate that the Lagrangian averaging methods currently being explored in wave–mean flow interaction (Andrews & McIntyre 1978) may eventually provide a firmer footing.

I am indebted to Dr J. A. Pyle and Miss J. Haigh for figure 12.

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